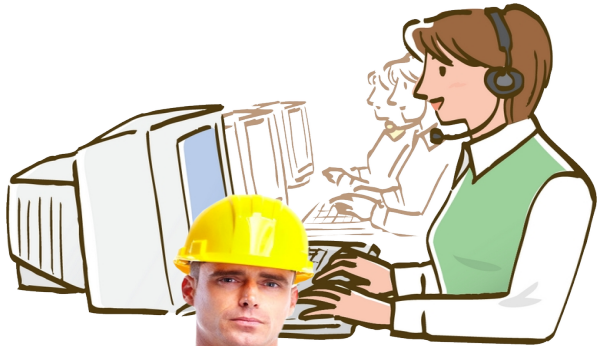
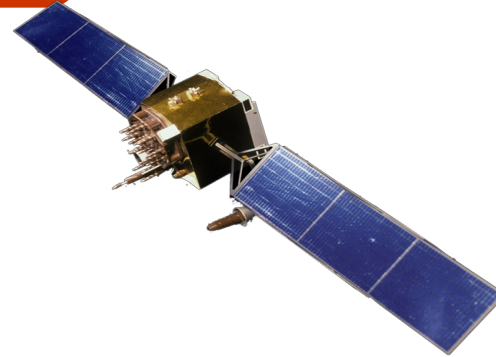


Chapter 3

Production and Cost

Production function



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Production Function

- An economic unit that produces goods or services is called a producer .
- The resources used for the production of goods are called inputs.
- Land, Labour, Capital and Organisation are called Factors of production.
- Goods and services produced using inputs are called output.
- Inputs are transformed in to output through the production process.

Production Function

Production function, in economics, equation that expresses the relationship between the quantities of productive factors (such as labour and capital) used and the amount of product obtained. It states the amount of product that can be obtained from every combination of factors, assuming that the most efficient available methods of production are used.

Suppose in a production function there are only two inputs, factor 1 and Factor 2 . If the amount of factor 1 X_1 and factor 2 is X_2 and output is q , The production function can be written as :

$$q = f (X_1, X_2)$$

Production Function

The relationship between inputs used and output produced is called production function

Process of Production

Land
Labour
Capital
Entrepreneurship



Car
Computers
Food
Medicines

Production Function table

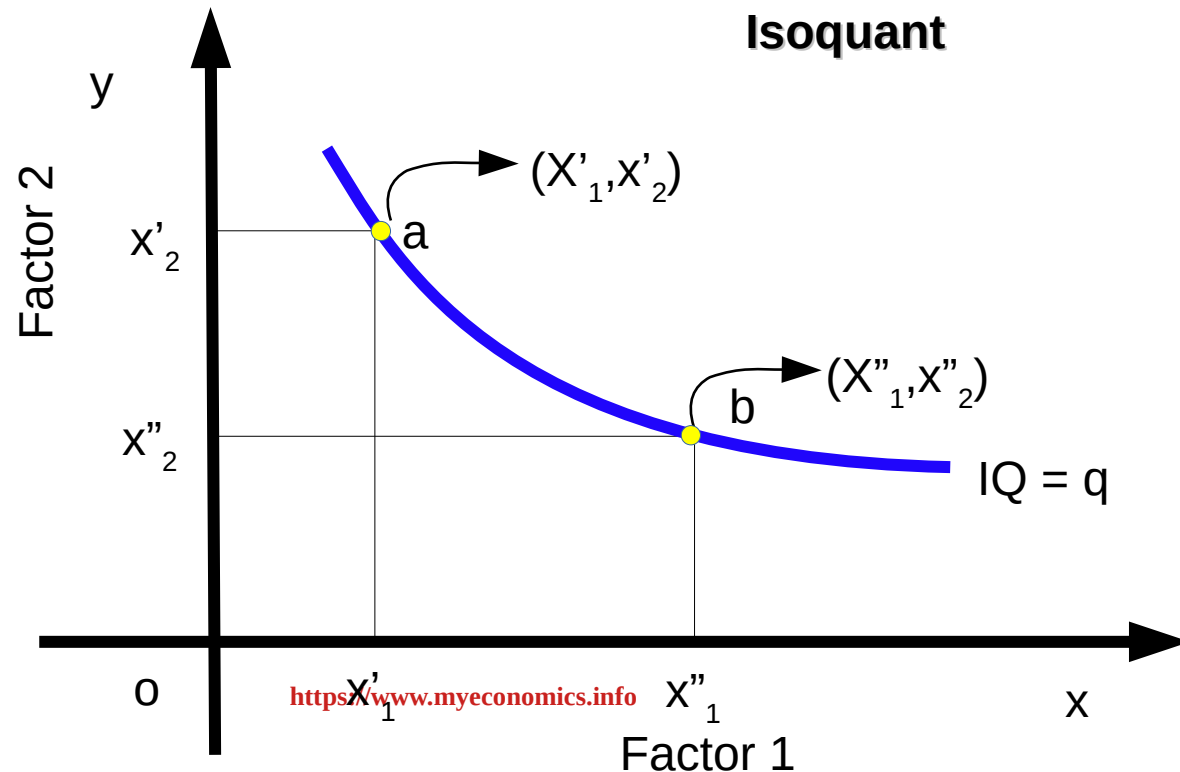
Factor 2		X_2				
		0	1	2	3	4
X_1	0	0	0	0	0	0
	1	0	1	3	7	10
	2	0	3	10	18	24
	3	0	7	18	30	40
	4	0	10	24	40	50
	5	0	12	29	46	56
	6	0	13	32	50	57

If input is zero, output is also zero. As input combinations change, quantity of output also change. As inputs increase, output also increase.

Isoquant

Isoquant can be defined as the locus of points of combinations of two inputs, which give the same Level of output.

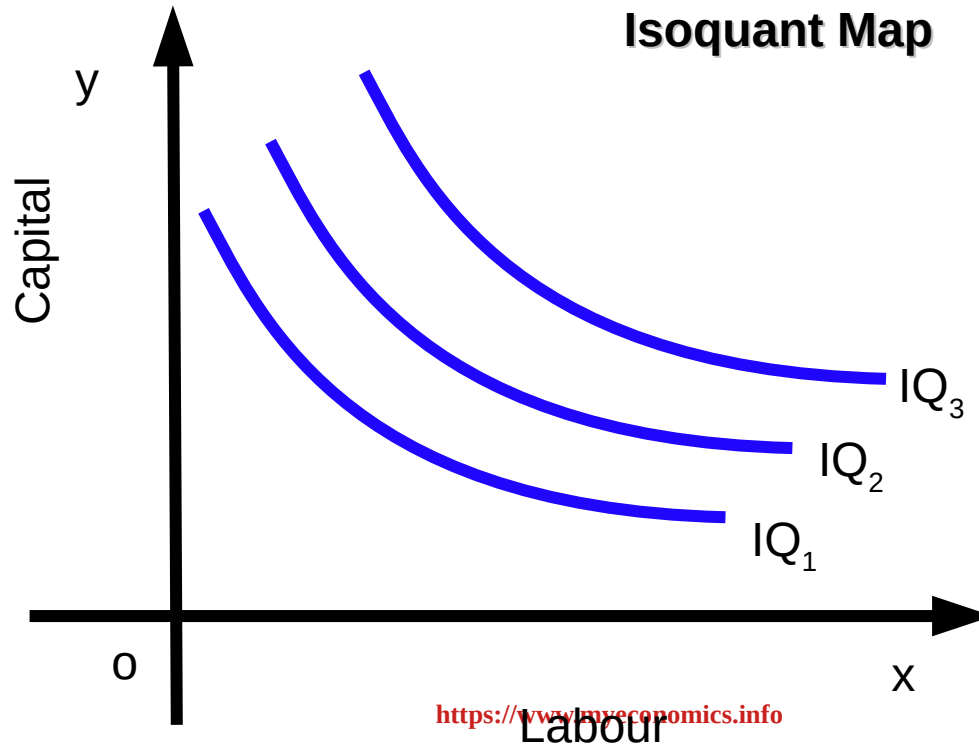
It is also known as **equal product curve**.



Isoquant

Isoquant can be defined as the locus of points of combinations of two inputs, which give the same Level of output.

It is also known as **equal product curve**.



A group of Isoquants is called an Isoquant map.

The Short Run and the Long Run

On the basis of the changes in input that a producer or firm can bring about time period is classified under two heads;

- **The Short run** and
- **The Long run**

The Short Run

Short run is the period in which units of some factors can be changed and others cannot be changed.

Eg: Increasing or decreasing the number of employees.



The Long Run

When a producer or a firm can bring about changes in all the inputs, that time period is called long run.

Eg: Remodelling factory, building or installing new machinery.



Inputs

On the basis of time periods inputs are classified under two categories -

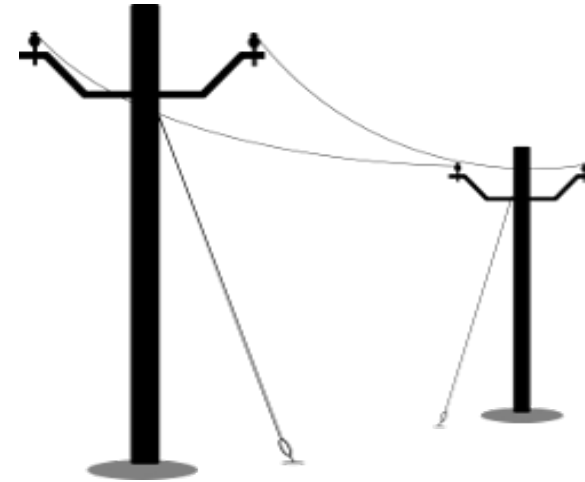
1). Variable Inputs

2). Fixed Inputs

Variable Inputs

Inputs that can bring about changes in the short run are called variable inputs.

Eg: Number of labour, raw materials, electricity etc.



Fixed Inputs

Short run inputs which can not be changed are called fixed inputs.

Eg: Size of building, installing new machinery...



Remember that..

The distinction between fixed inputs and variable inputs is valid only in the short run. In the long run all inputs are variable.

Total Product

Total Product or total physical product is the total output produced when a change in quantity of one variable input alone is affected while all other inputs remain constant.

Total Product

Total product can be represented as a relation between variable inputs and output.

$$q = f(x_1 : \bar{x}_2)$$

Here,

q = total product, x_1 = variable input,

\bar{x}_2 = fixed input.

Average Product

Average product is the output per unit of variable input factor.

We can find AP by using the below equation,

$$AP_1 = \frac{TP}{x_1} = \frac{f(x_1 : \bar{x}_2)}{x_1}$$

Average Product

Variable factor inputs	TP	AP
0	0	?
1	1	?
2	3	?
3	7	?
4	10	?
5	12	?
6	13	?

Average Product

Activity:1 An:

Variable factor 1 inputs (x_1)	TP	AP
0	0	∞
1	1	$\frac{1}{1} = 1$
2	3	$\frac{3}{2} = 1.5$
3	7	$\frac{7}{3} = 2.33$
4	10	$\frac{10}{4} = 2.5$
5	12	$\frac{12}{5} = 2.4$
6	13	$\frac{13}{6} = 2.16$

Average Product

Activity: 2

Variable factor 1 inputs (x_1)	(AP)	TP
1	10	?
2	12	?
3	13	?
4	14	?
5	14	?
6	13	?
7	12	?

<https://www.myeconomics.info>

Average Product

Ans:

Variable factor 1 inputs (x_1)	AP	TP = $x_1 \times$ AP
1	10	1×10=10
2	12	2×12=24
3	13	3×13=39
4	14	4×14=56
5	14	5×14=70
6	13	6×13=78
7	12	7×12=84

Marginal Product

'Marginal Product is the addition made to the total product through the addition of one unit of a variable input, keeping all other inputs constant.'

Marginal Product

$$MP_1 = \frac{\text{Change in TP}}{\text{Change in variable input}} = \frac{\Delta q}{\Delta X_1}$$

Marginal Product

If change in input is discrete units MP can be defined as follows -

$$MP_1 = f(x_1 : \bar{x}_2) - f(x_1 - 1 : \bar{x}_2)$$

= (TP when X_1 is the unit) - (TP when $X_1 - 1$ is the unit)

$$TP_n = MP_1 + MP_2 + \dots + MP_n = \sum_{i=1}^n MP_i$$

$$AP_n = \frac{MP_1 + MP_2 + \dots + MP_n}{n} = \frac{\sum_{i=1}^n MP_i}{n}$$

Marginal Product

Qn.1). Find AP and MP on the basis of TP

Input	TP
0	0
1	1
2	3
3	6
4	10
5	15
6	15
7	14
8	12

Marginal Product

An.1

$$\text{MP} = (\text{TP when } X_1 \text{ is the unit}) - (\text{TP when } X_1 - 1 \text{ is the unit})$$

Input	TP	MP
0	0	---
1	1	$1 - 0 = 1$
2	3	$3 - 1 = 2$
3	7	$7 - 3 = 4$
4	10	$10 - 7 = 3$
5	12	$12 - 10 = 2$
6	13	$13 - 12 = 1$

Marginal Product

Qn.2). Find TP and AP on the basis of MP

Input	MP
0	7
1	11
2	12
3	10
4	6
5	4
6	2
7	0
8	-2

Marginal Product

An:

Input	MP	TP	AP
0	7	7	--
1	11	18	18
2	12	30	15
3	10	40	13.3
4	6	46	11.5
5	4	50	10
6	2	52	8.66
7	0	52	7.43
8	-2	50	6.25

Law of Diminishing Marginal Product Or

Law of Variable Proportions.

- It is a short run production function
- It explains the effect of change in total output when one variable input is changed and other inputs kept constant
- It is also called returns to a factor

Law of Diminishing Marginal Product Or

Law of Variable Proportions.

Law of Variable proportion states that, “when more and more units of a variable factor are added with fixed factors; Initially TP increases at an increasing rate, then increases at a diminishing rate and finally starts to decline”.

Law of Diminishing Marginal Product Or

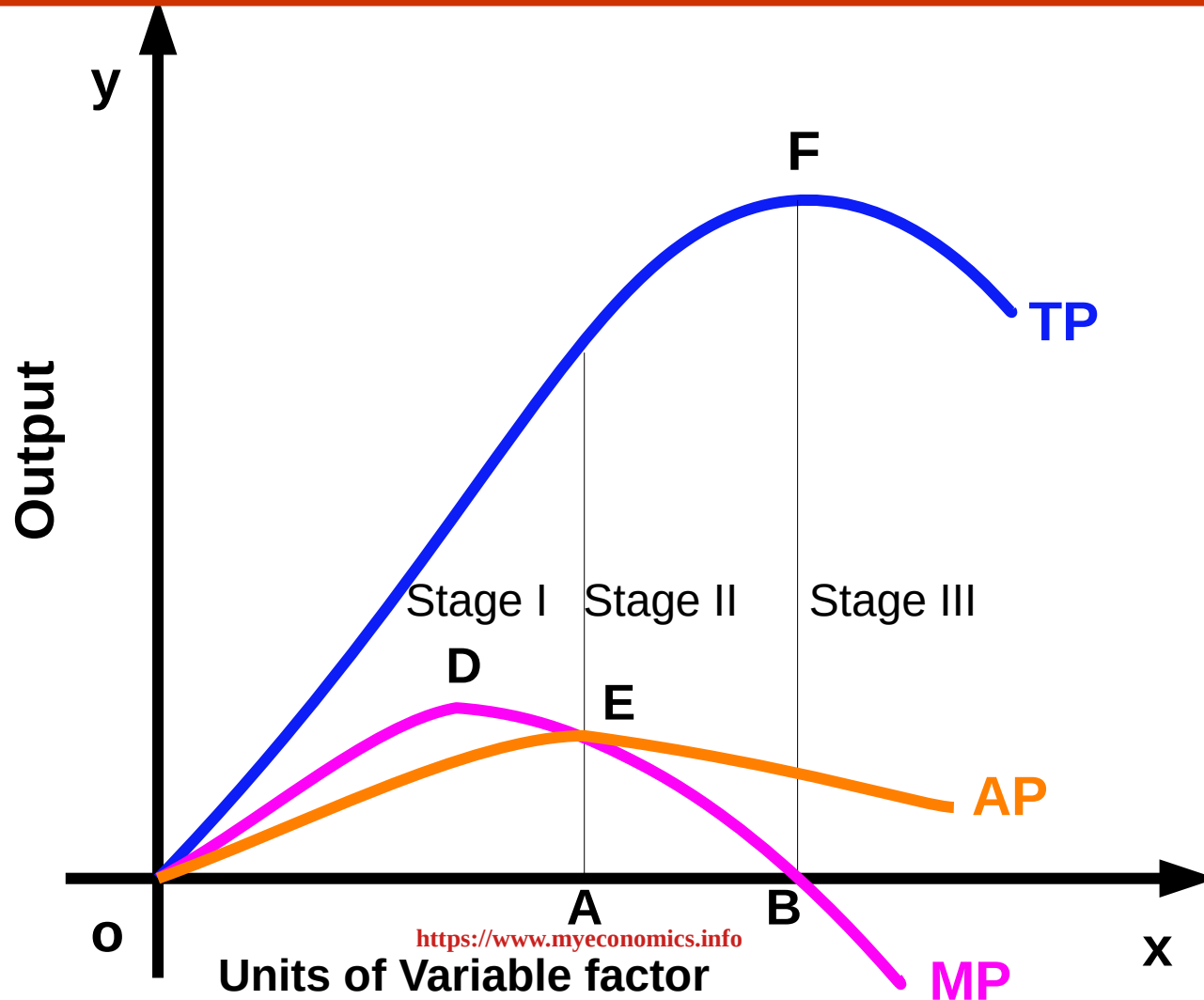
Law of Variable Proportions.

When more units of variable factors are added to constant factors, the relationship between them or proportion changes. This is why the law is called the **Law of Variable Proportion.**

Short run production function table

Fixed Input X_2	Variable Input X_1	Total Product	Marginal product	Average Product
4	0	0	-	-
4	1	1	1	1
4	2	3	2	1.5
4	3	6	3	2
4	4	10	4	2.5
4	5	13	3	2.6
4	6	15	2	2.5
4	7	16	1	2.3
4	8	16	0	2
4	9	15	-1	1.6
4	10	13	-2	1.3

Short run production function-graph



When more and more units of variable factors are added to fixed factors, Total product (TP) And Marginal Product (MP) pass through three stages.

Stage 1

Increasing Returns to a factor

Stage 2

Diminishing Returns to a factor

Stage 3

Negative Returns to a factor

When more and more units of variable factors are added to fixed factors, Total product (TP) and Marginal Product (MP) pass through three stages.

Stage 1

Increasing Returns to a factor

TP increases at an increasing rate.
MP increases and reaches maximum.
AP also increases.

When more and more units of variable factors are added to fixed factors, Total product (TP) and Marginal Product (MP) pass through three stages.



Stage 2

Diminishing Returns to a factor



TP increases at a decreasing rate.
MP falls but remains positive. AP declines.

When more and more units of variable factors are added to fixed factors, Total product (TP) and Marginal Product (MP) pass through three stages.

Stage 3

Negative Returns to a factor

TP starts declining. MP becomes negative.

This is Rajeev. He is a farmer who produces paddy in his 10 acres of land. He is trying to increase his output by increasing the quantity of only one input-labour. What will be the effect on his total output?

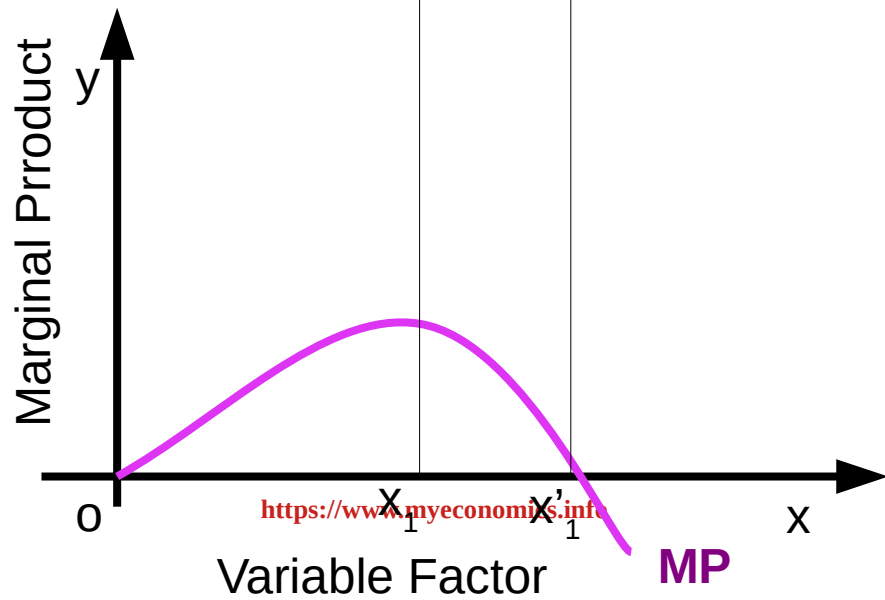
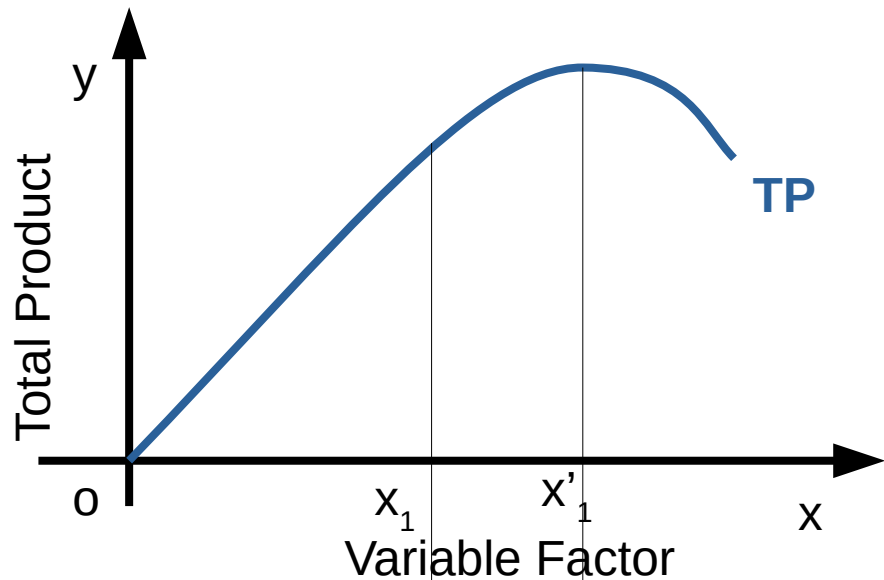


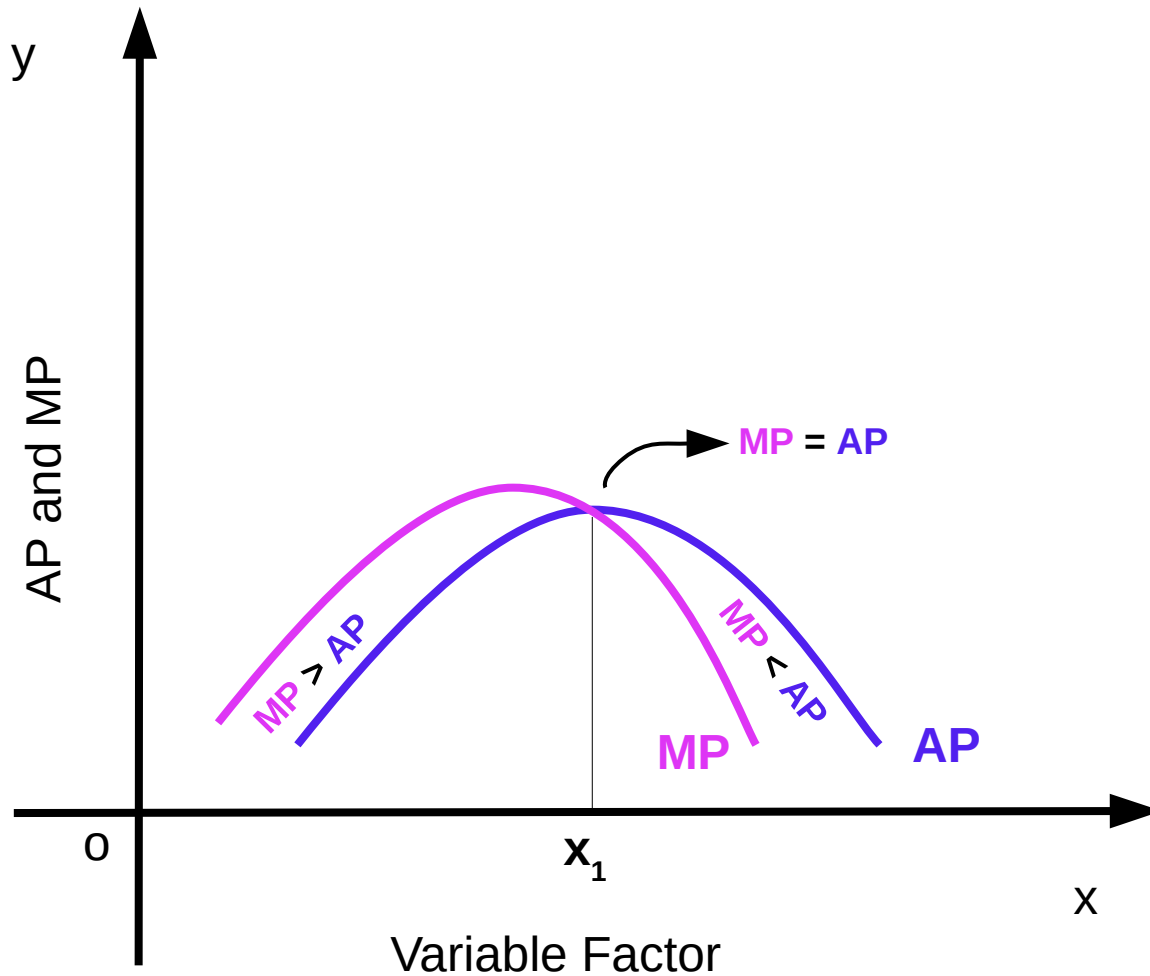
As the employment of variable factor labour increases, by keeping all other inputs constant, the factor proportions initially become more suitable for production. Therefore initially Marginal Product increases. But after the first stage the relation between fixed and variable factors become less suitable for production and MP decreases.



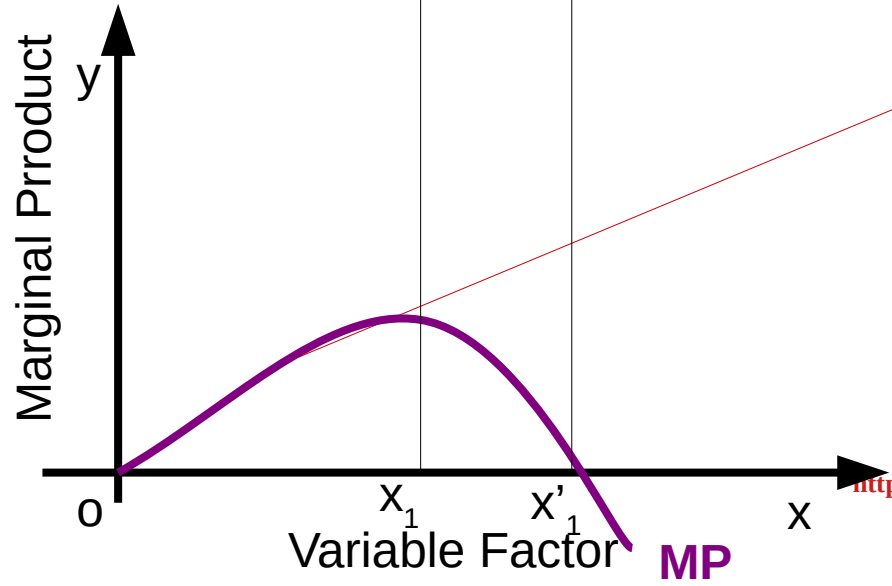
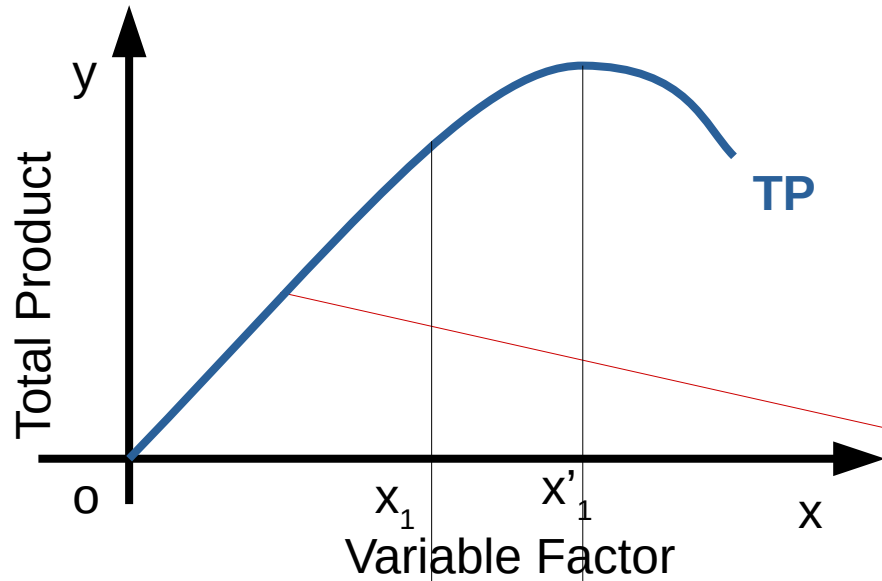
Short run production function table

Fixed Input X_2	Variable Input X_1	Total Product	Marginal product	Average Product
4	0	0	-	-
4	1	1	1	1
4	2	3	2	1.5
4	3	6	3	2
4	4	10	4	2.5
4	5	13	3	2.6
4	6	15	2	2.5
4	7	16	1	2.3
4	8	16	0	2
4	9	15	-1	1.6
4	10	https://www.myeconomics.info 13	-2	1.3

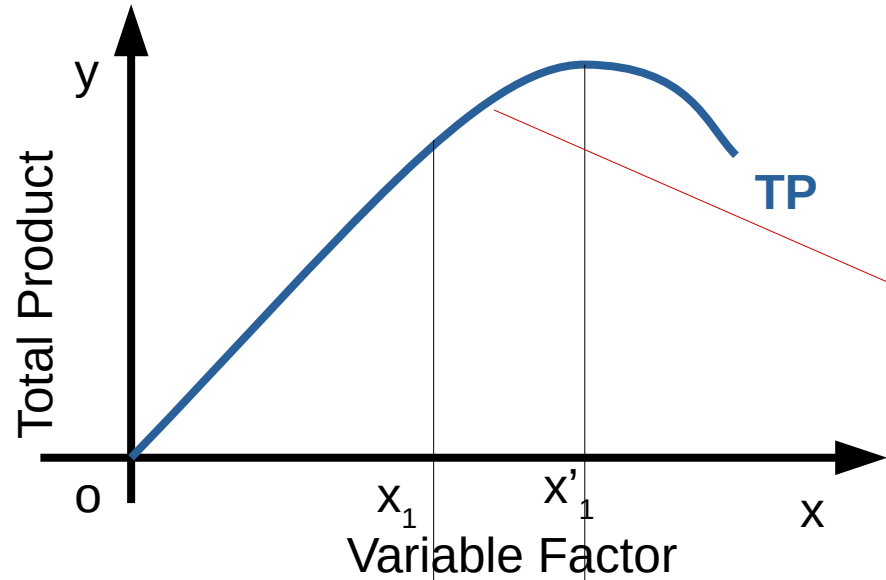




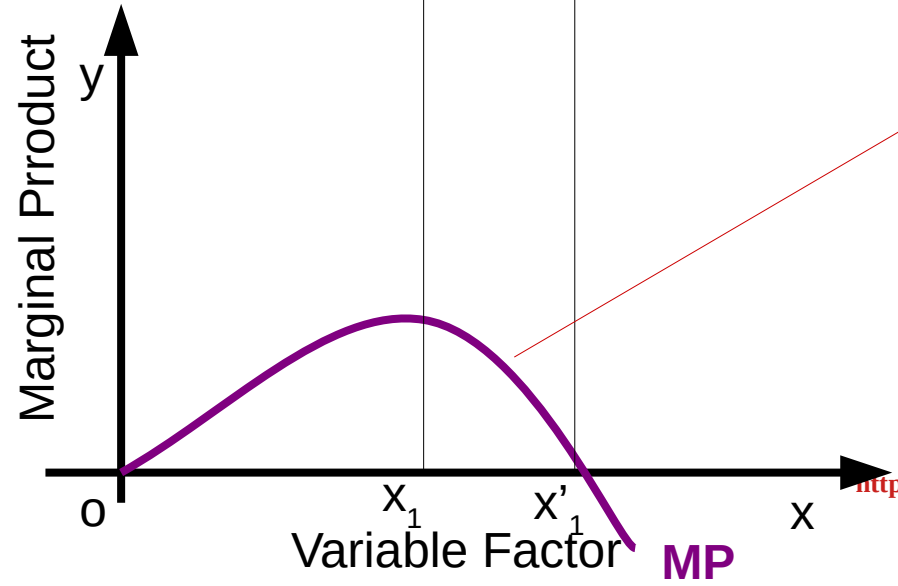
Relation between TP and MP

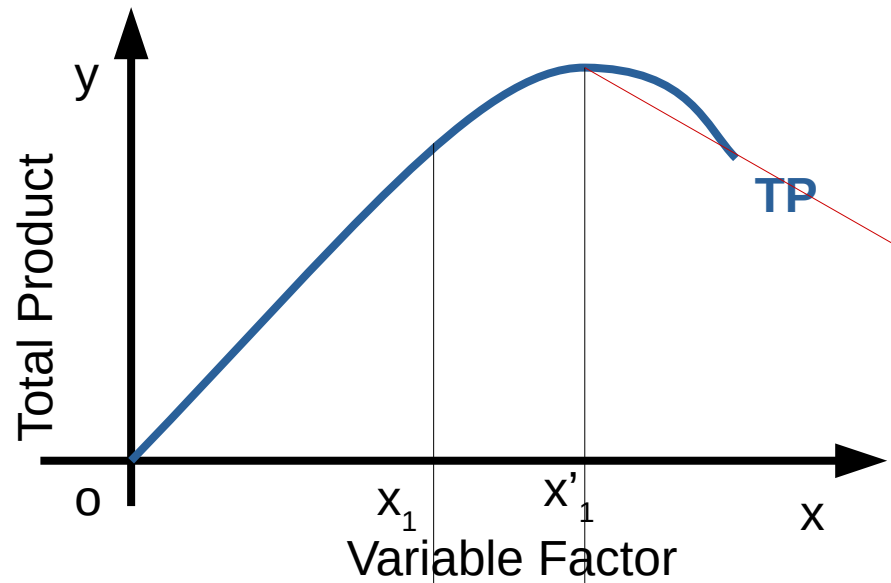


When TP increases at an increasing rate MP increases.

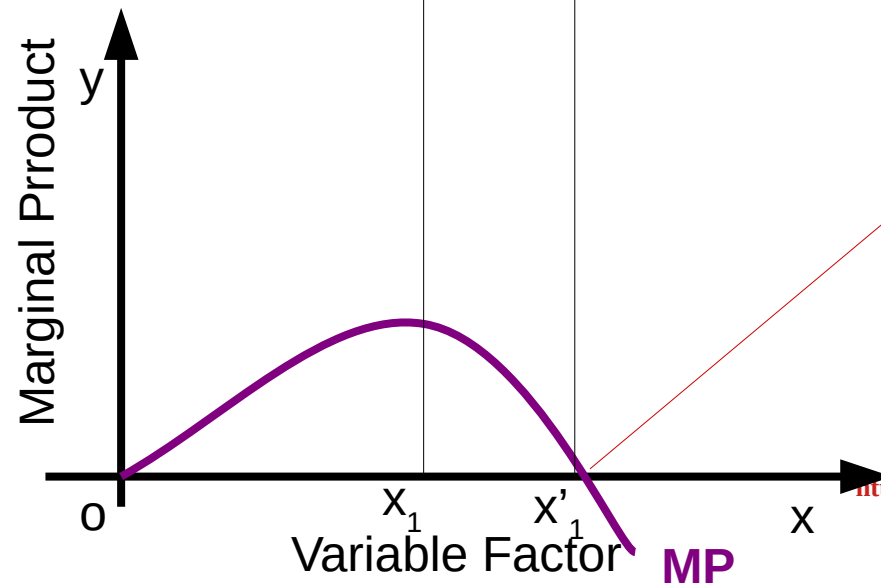


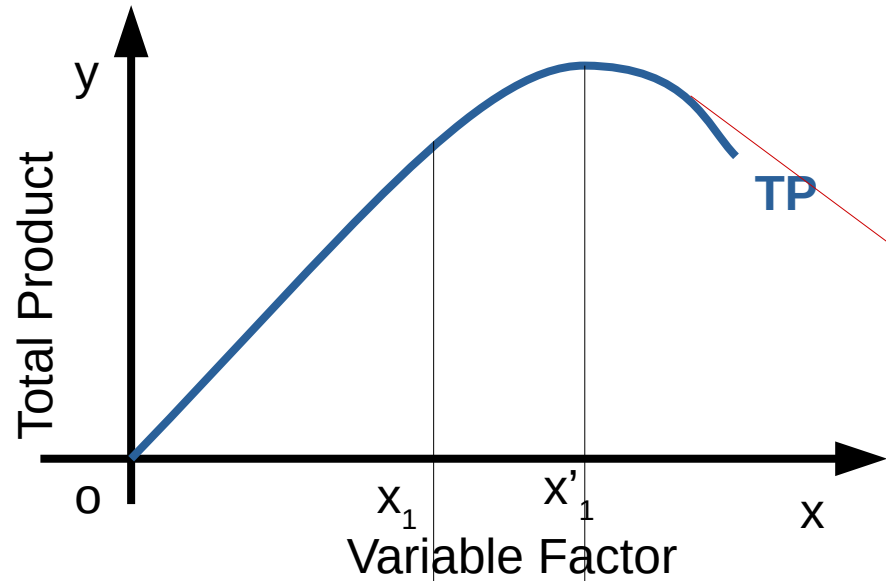
When TP increases at a decreasing rate MP decreases but remains positive.



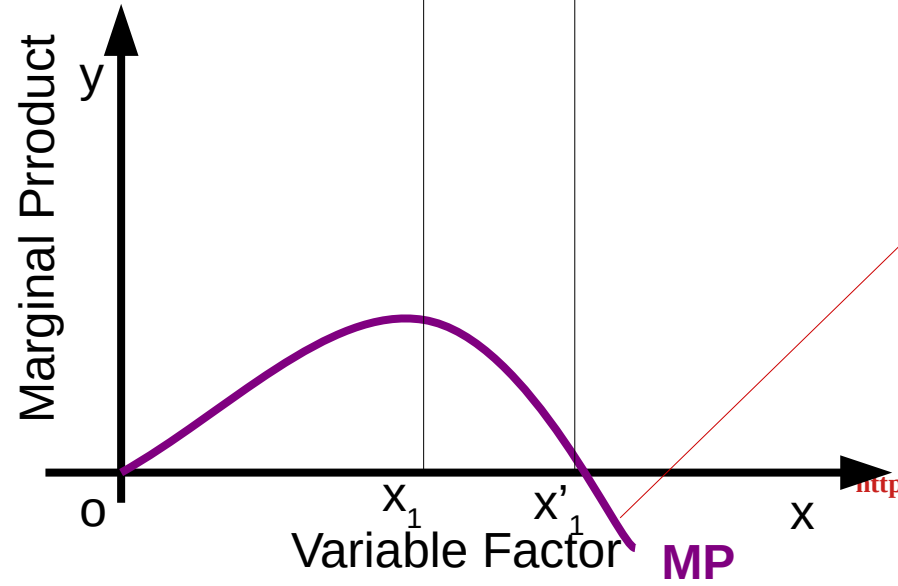


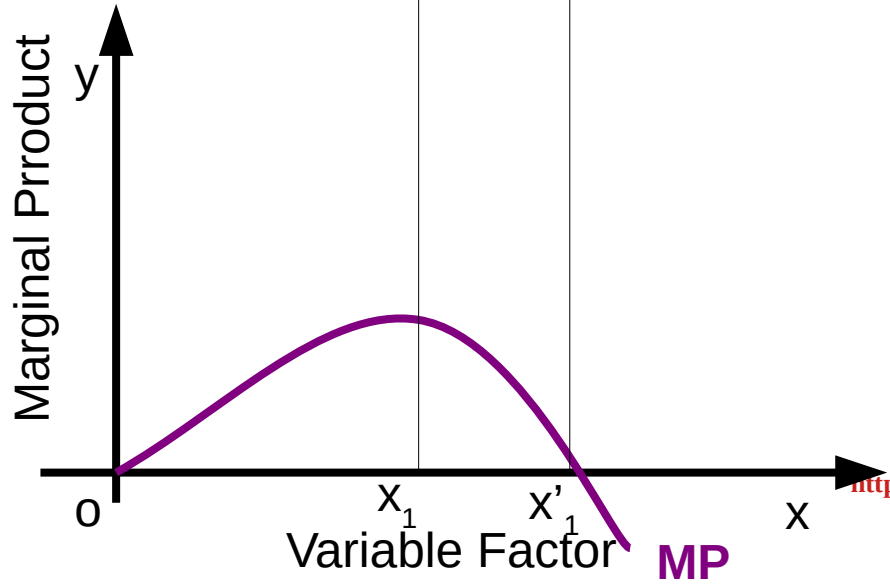
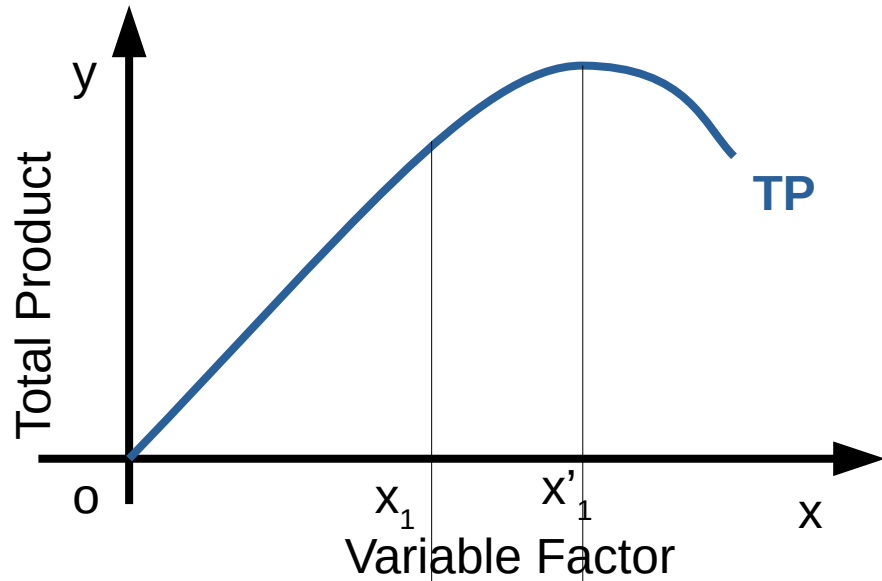
When TP reaches its maximum MP becomes zero.



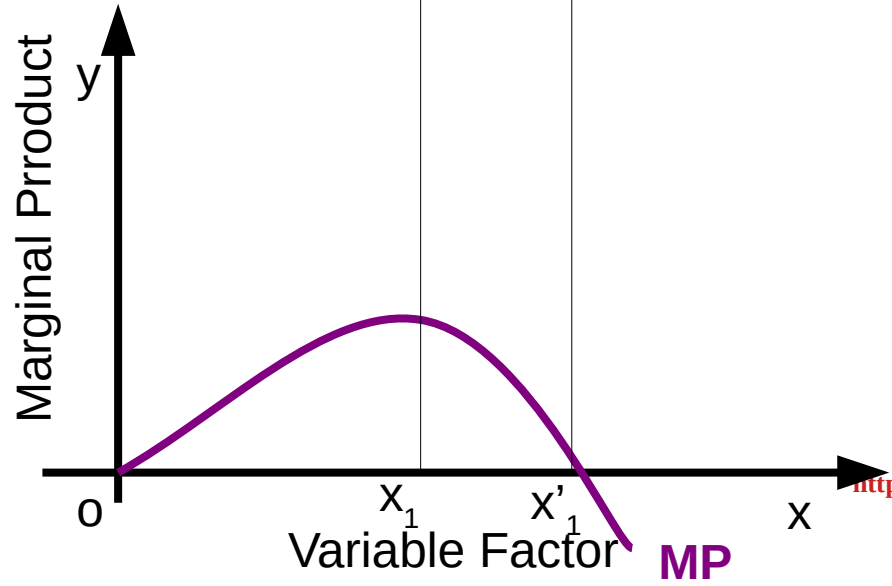
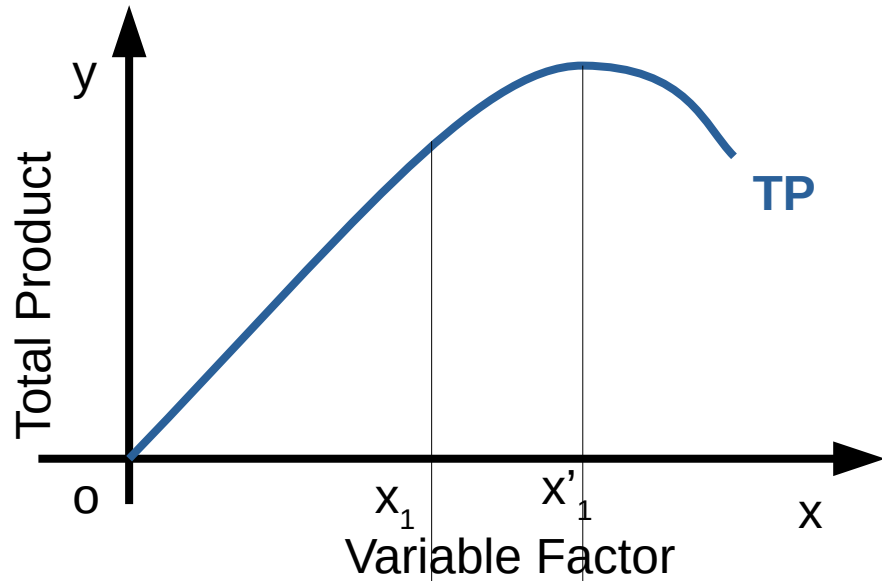


When TP decreases, MP becomes negative .



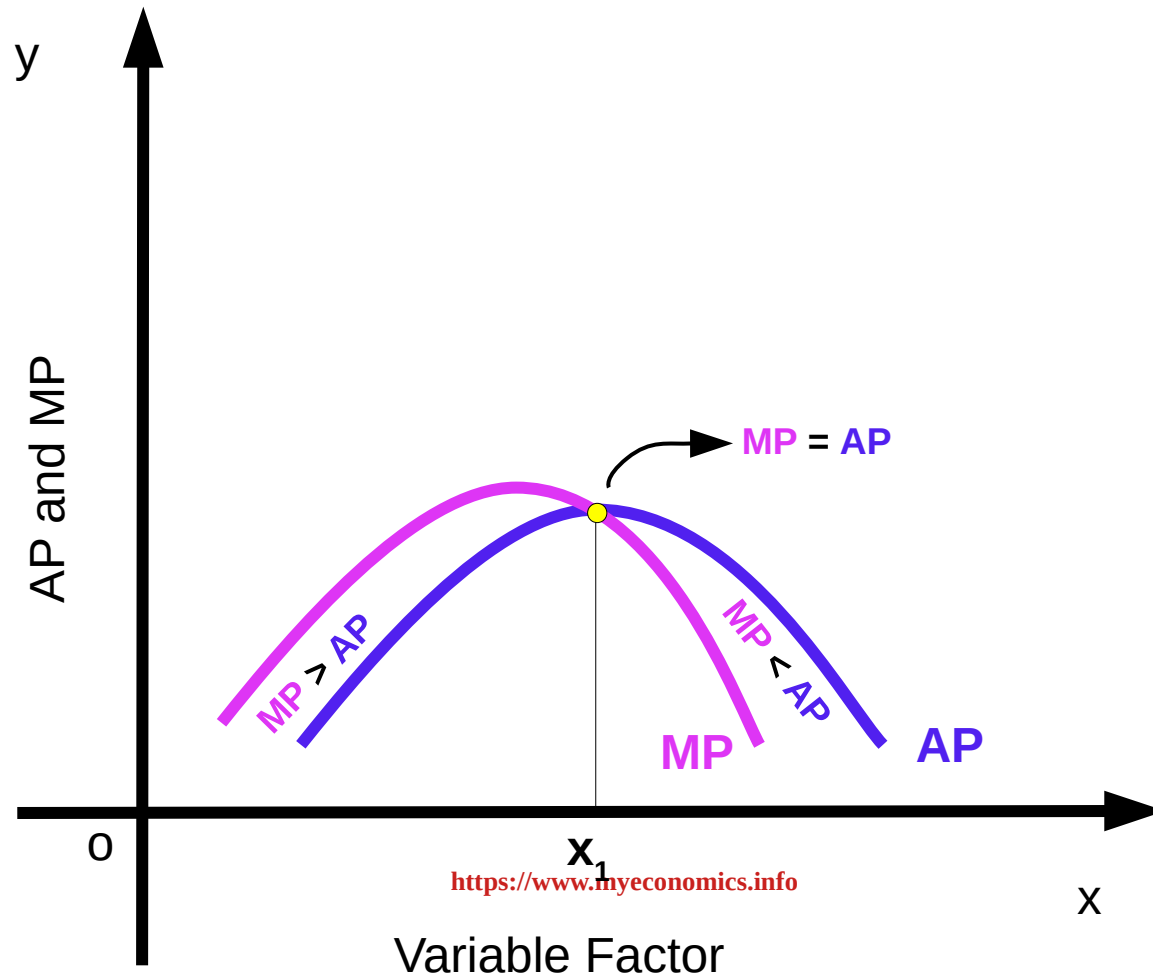


When TP increases at an increasing rate MP increases. When TP increases at a decreasing rate MP decreases but remains positive. When TP reaches its maximum MP becomes zero. When TP decreases, MP becomes negative.

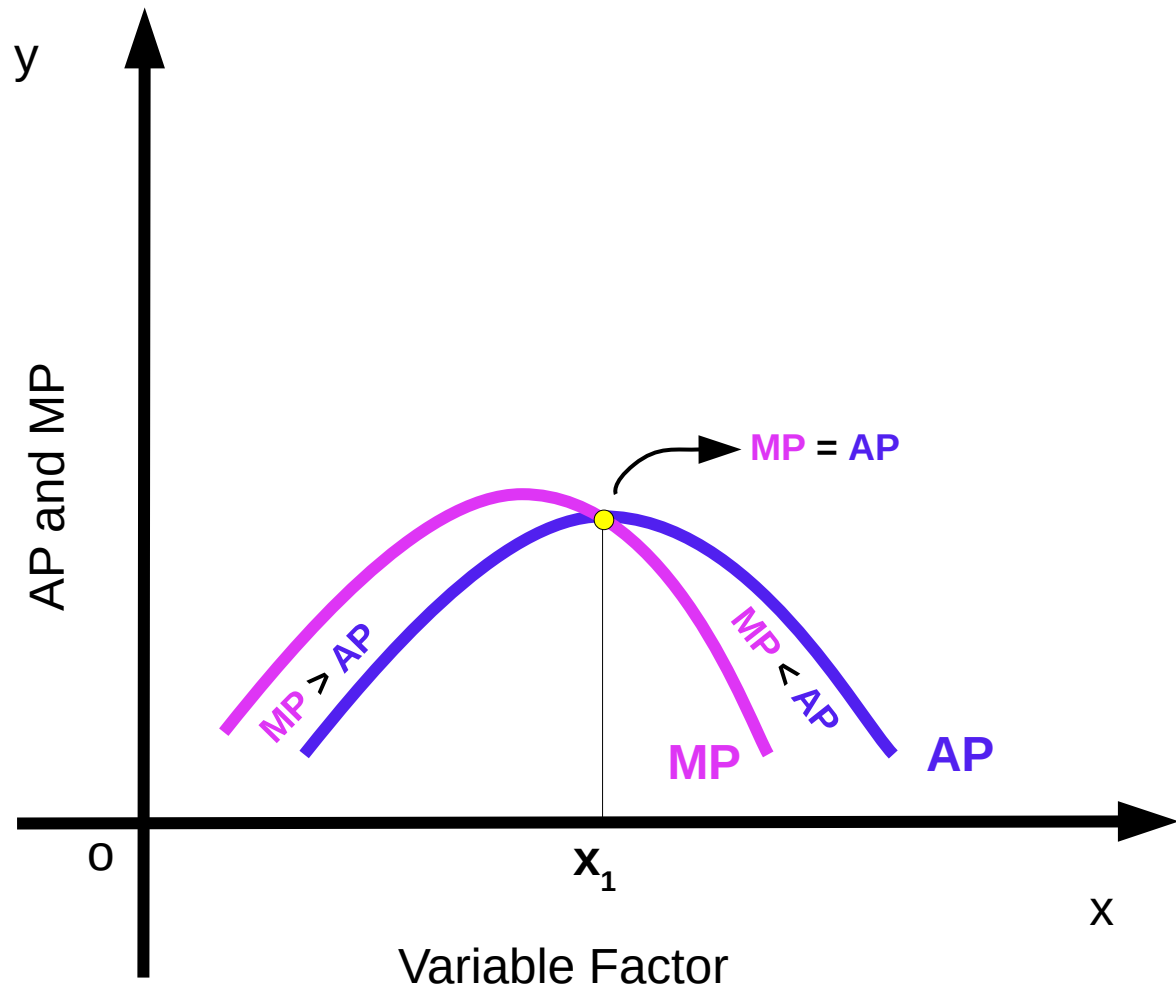


When TP increases at an increasing rate MP increases. When TP increases at a decreasing rate MP decreases but remains positive. When TP reaches its maximum MP becomes zero. When TP decreases, MP becomes negative.

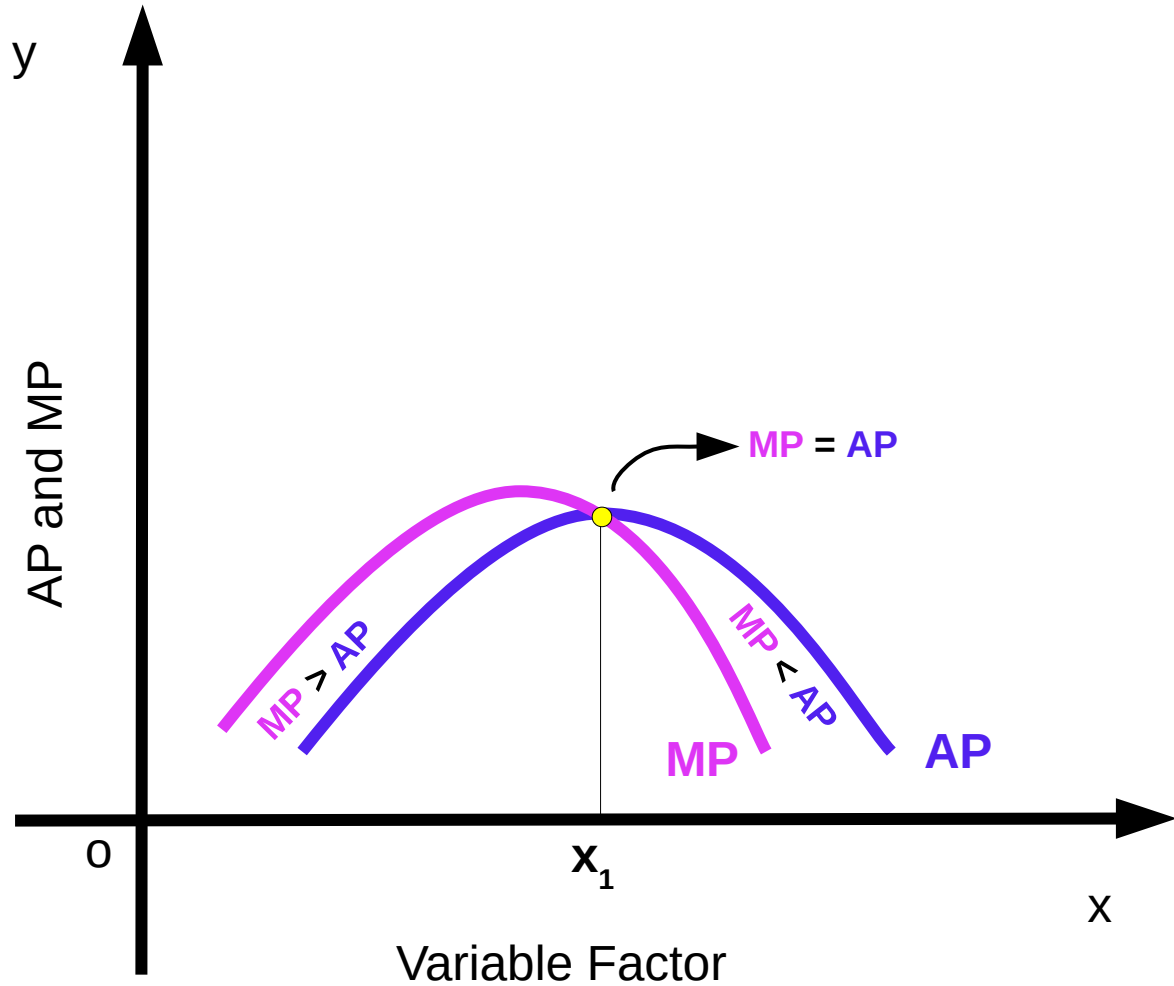
Relation between AP and MP



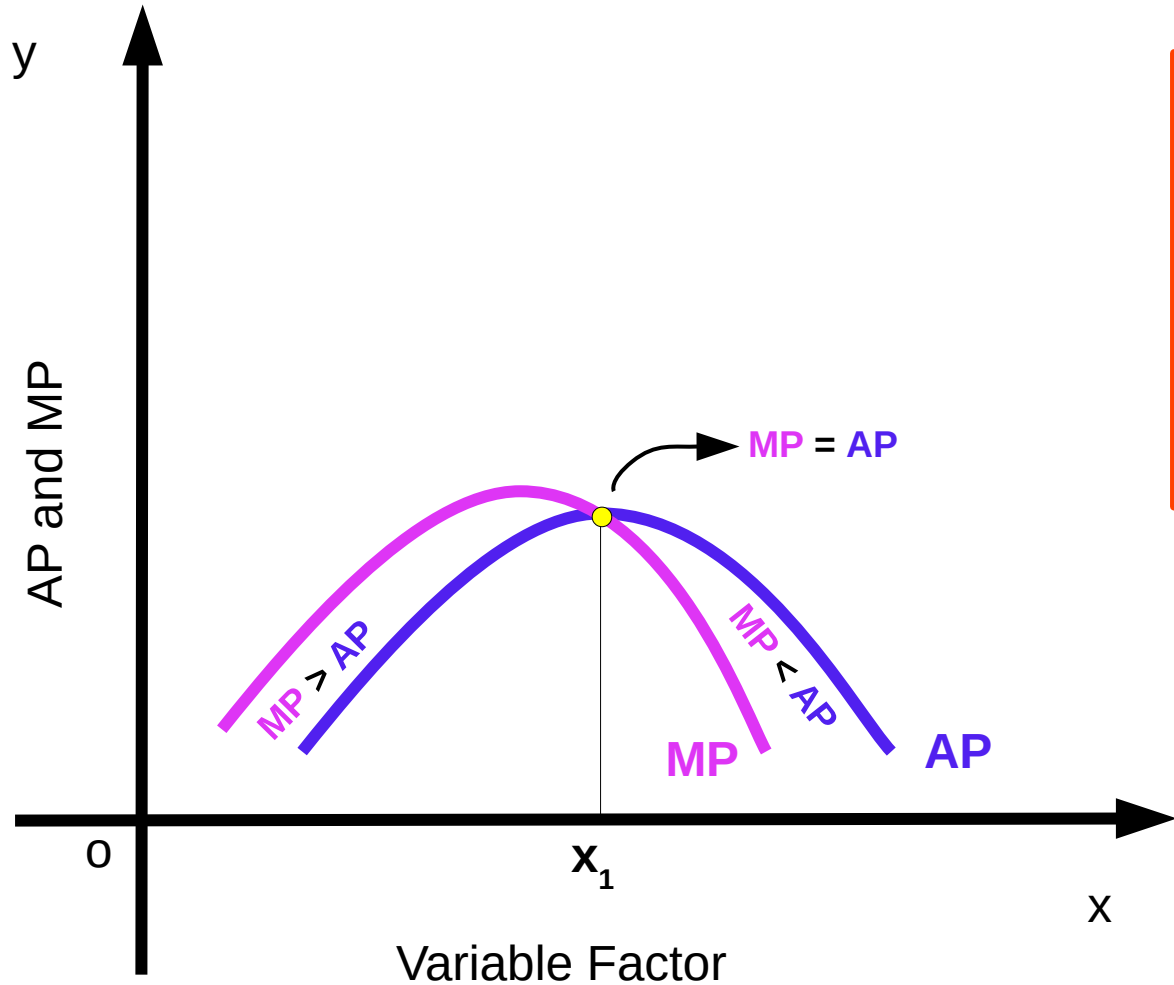
Relation between AP and MP



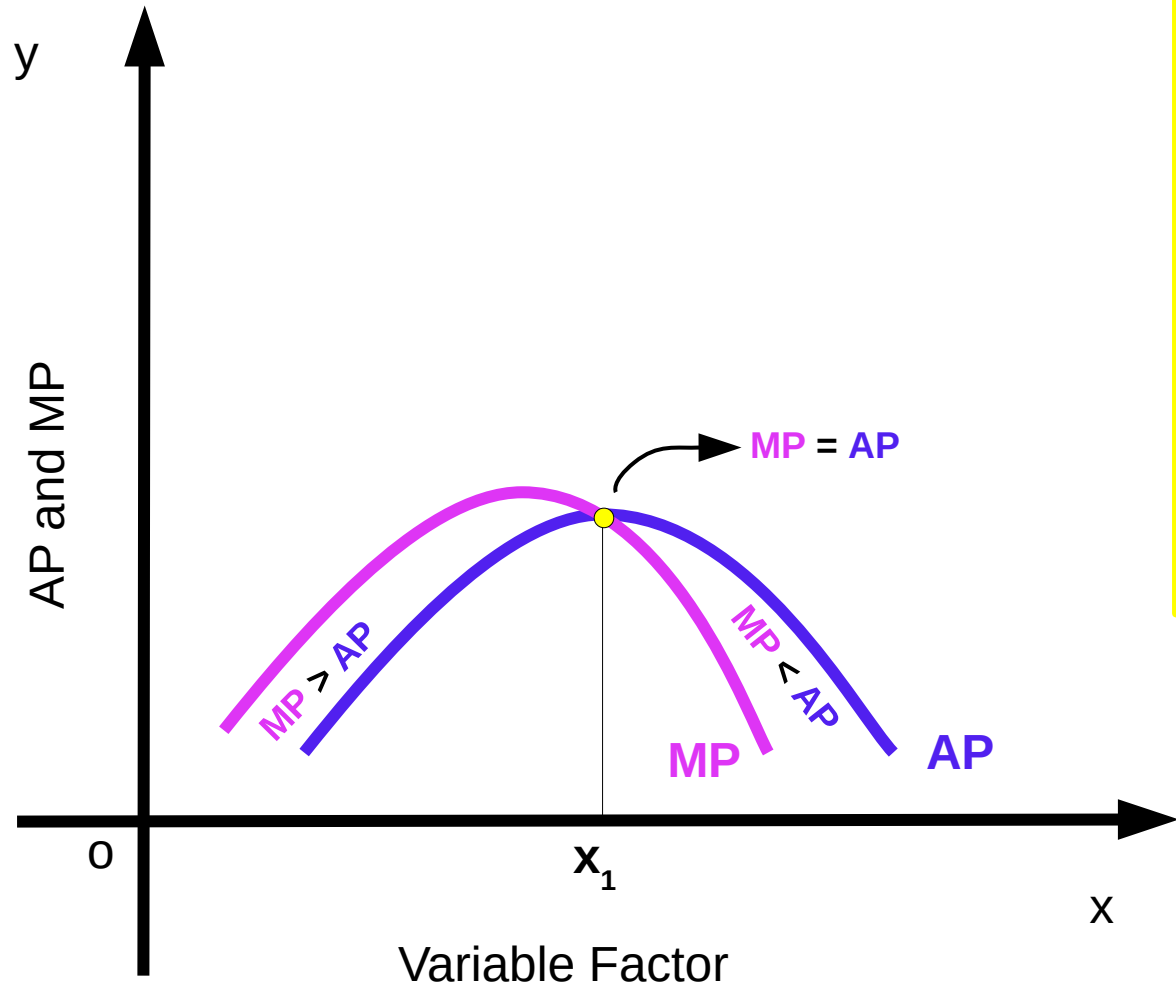
As AP increases, MP is greater than AP. Then MP curve is above the AP curve.



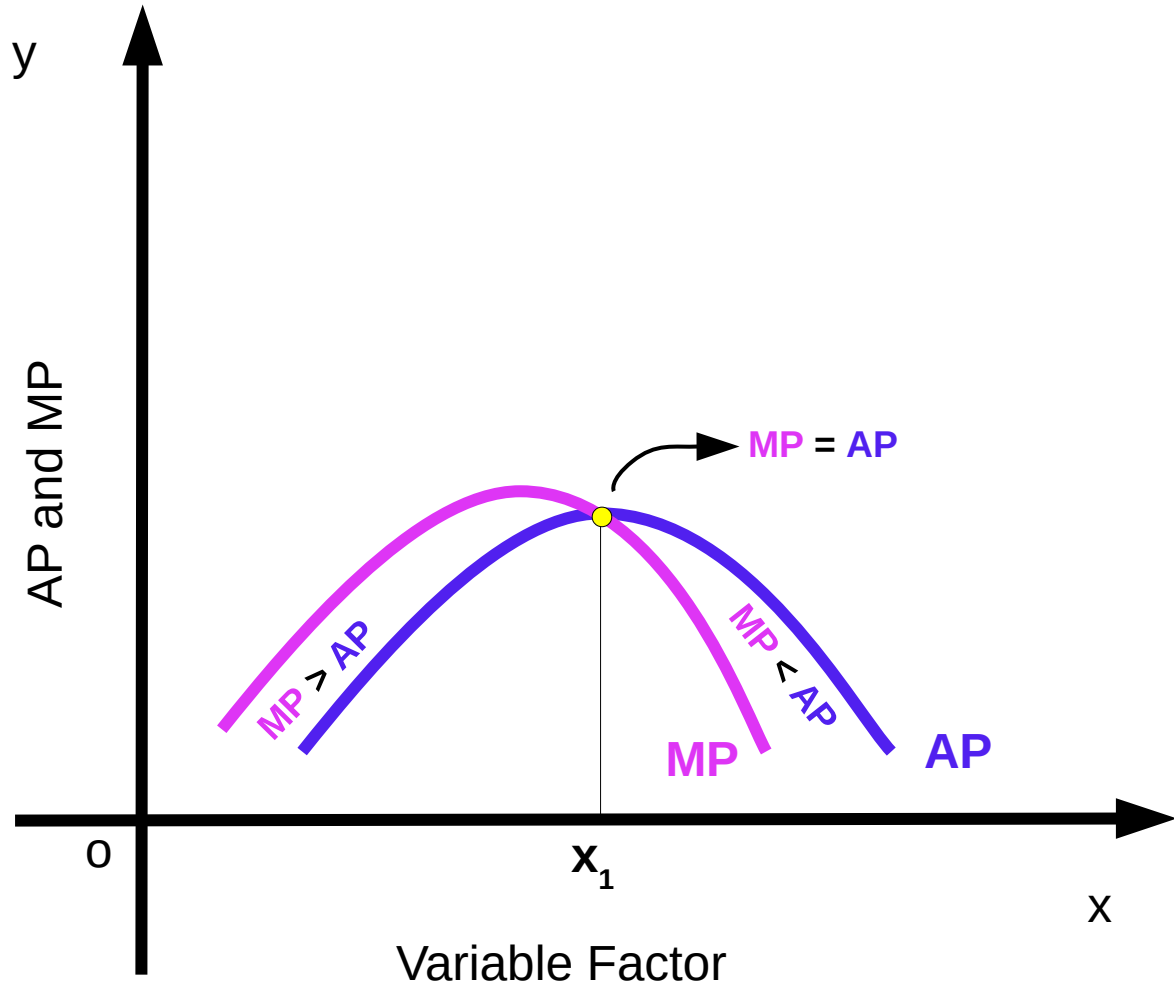
When AP reaches maximum,
AP and MP will be equal.



when AP decreases, MP is less than AP. Then MP curve is below the AP curve.



As AP increases, MP is greater than AP. Then MP curve is above the AP curve. When AP reaches maximum, AP and MP will be equal. when AP decreases, MP is less than AP. Then MP curve is below the AP curve.



AP curve and MP curve have inverted U shape. AP will never be zero or negative. But MP can be zero and negative.

Returns to scale

- It is a long run production function.
- In the long run all factors are variable.
- Returns to scale refers to changes in output caused by proportionate change in all inputs.
- The input ratios remain the same.

Returns to scale

The effect of change in output with respect to proportionate change in all inputs is called long run production function.

When all inputs are changed in the same proportion, Total Product responds in three different ways. They are:



Increasing Returns to Scale- IRS

Constant Returns to Scale- CRS

Diminishing Returns to Scale- DRS

Returns to scale

Increasing Returns to Scale- IRS

A proportionate increase in all inputs result in more than proportionate increase in output.

When the inputs are increased by 100%, the output is increased by more than 100%, the Returns to scale is IRS

Factor 1 (x_1)	Factor 1 (x_2)	Ratio (x_1)(x_2)	TP	Increase in input (%)	Increase in output (%)
5	5	5:5	20	--	--
10	10	5:5	50	100	150
20	20	5:5	150	100	200

Returns to scale

Constant Returns to Scale- CRS

A proportionate increase in all inputs result in an equal proportionate increase in output.

When the inputs are increased by 100%, the output is also increased by 100%, the Returns to scale is CRS

Factor 1 (x_1)	Factor 1 (x_2)	Ratio (x_1)(x_2)	TP	Increase in input (%)	Increase in output (%)
5	5	5:5	20	--	--
10	10	5:5	40	100	100
20	20	5:5	80	100	100

Returns to scale

Diminishing Returns to Scale- DRS

A proportionate increase in all inputs result in a less than proportionate increase in output.

When the inputs are increased by 100%, the output is increased by less than 100%, the Returns to scale is DRS

Factor 1 (x_1)	Factor 1 (x_2)	Ratio (x_1)(x_2)	TP	Increase in input (%)	Increase in output (%)
5	5	5:5	20	--	--
10	10	5:5	30	100	50
20	20	5:5	42	100	40

Returns to scale

Increasing Returns to Scale- IRS

A proportionate increase in all inputs result in more than proportionate increase in output.

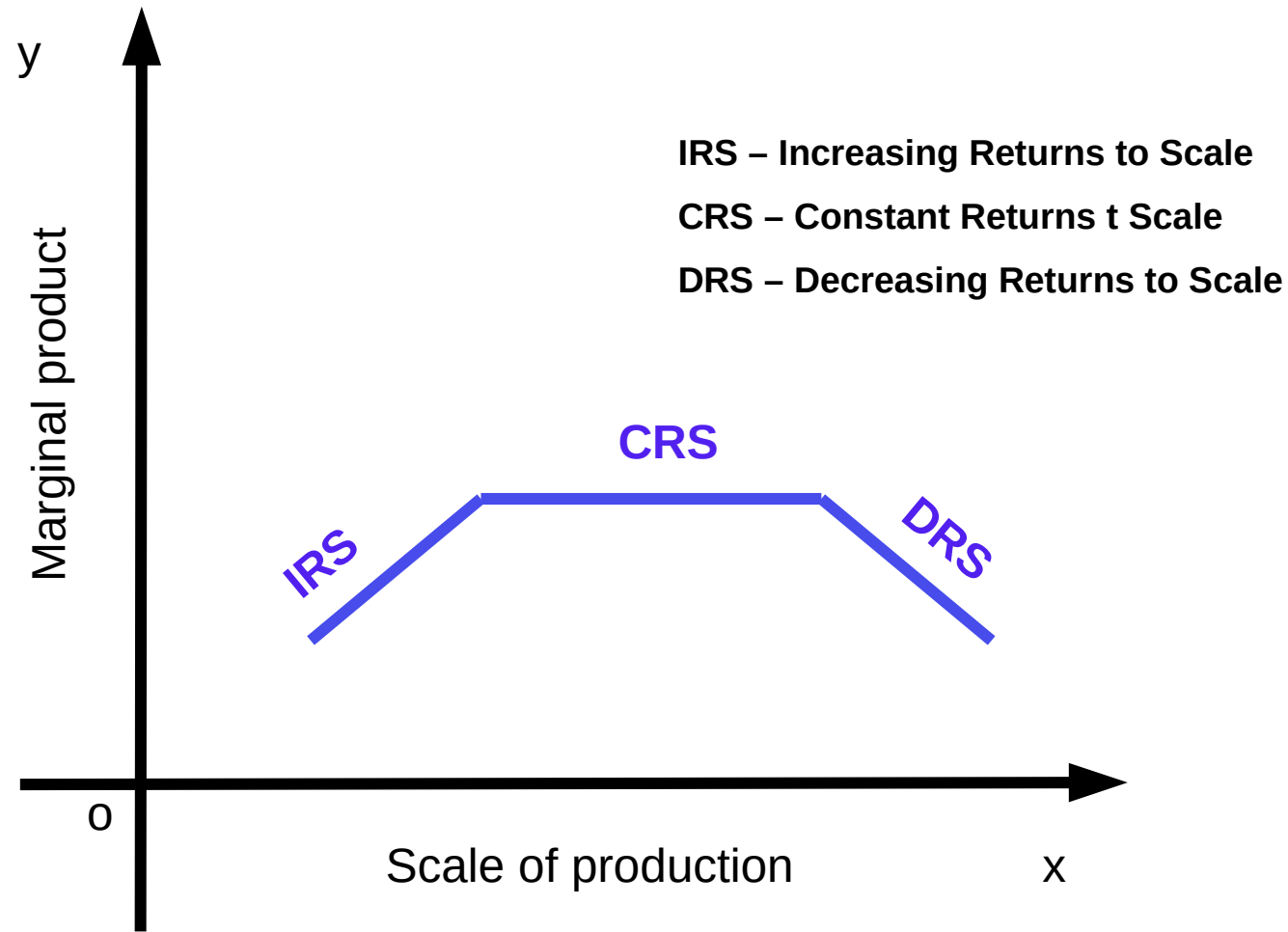
Constant Returns to Scale- CRS

A proportionate increase in all inputs result in an equal proportionate increase in output.

Diminishing Returns to Scale- DRS

A proportionate increase in all inputs result in a less than proportionate increase in output.

Returns to Scale - graph



Find the percentage increase of input and total product and mark IRS, CRS and DRS on the below given table.

Scale	Input		Total Product q	Percentage increase			Laws of Return
	Capital K	Labour L		Capital K	Labour L	Total Product q	
A	8	10	100	-	-	-	?
	12	15	180	?	?	?	
B	8	4	100	-	-	-	?
	10	5	125	?	?	?	
C	10	20	200	-	-	-	?
	14	28	260	?	?	?	

Solution:

Scale	Input		Total Product q	Percentage increase			Laws of Return
	Capital K	Labour L		Capital K	Labour L	Total Product q	
A	8 12	10 15	100 180	$\frac{12-8}{8} \times 100 = 50$	$\frac{15-10}{10} \times 100 = 50$	$\frac{180-100}{100} \times 100 = 80$	IRS
B	8 10	4 5	100 125	$\frac{10-8}{8} \times 100 = 25$	$\frac{5-4}{4} \times 100 = 25$	$\frac{125-100}{100} \times 100 = 25$	CRS
C	10 14	20 28	200 260	$\frac{14-10}{10} \times 100 = 40$	$\frac{28-20}{20} \times 100 = 40$	$\frac{260-200}{200} \times 100 = 30$	DRS

Cobb Douglas Production function

Cobb Douglas production function is a linear homogeneous Production function.

That means, If factors are increased 't' times, output will also increase 't' times, where ($t > 1$)

If $\alpha + \beta = 1$, production function is **CRS**

If $\alpha + \beta > 1$, production function is **IRS**

If $\alpha + \beta < 1$, production function is **DRS**

Cobb Douglas Production function

C. W. Cobb and Paul H Douglas formulated this Production Function

It is represented as

$$q = x_1^\alpha x_2^\beta$$

Where,

Q = quantity of output

X_1 is the quantity of factor 1

X_2 is the quantity of factor 2

α And β are constants

<https://www.myeconomics.info>



Paul H Douglas

Cobb Douglas Production function

Suppose $q = 3L^4K^2$, if $L = 2, K = 5$

1. What is the output?
2. Which is the law of returns to scale?

Cobb Douglas Production function

Suppose $q = 3L^4K^2$, if $L = 2, K = 5$

1. What is the output?
2. Which is the law of returns to scale?

Answer : $q = 3L^4K^2$, if $L = 2, K = 5$

1. $q = 3 \times 2^4 \times 5^2$
 $= 3 \times 16 \times 25$
 $= 1200$

2. $4 + 2 = 6$, It is an IRS production function

COSTS

Costs refers to the expenses incurred by the producer to produce goods and services.

COSTS

What are the costs ?

Wage, rent, insurance premium, electricity charge, transportation cost, training cost etc are examples for costs.



COST FUNCTION

The relation between cost and product is called the cost function.

If $c = \text{cost}$ and $q = \text{product}$,

$$c = f(q)$$

COSTS

**In the short run, costs can be classified as total fixed cost and total variable cost.
In the long run all costs are variable.**

COSTS

Total Fixed Cost – TFC

The costs incurred by the producer to buy fixed inputs is called total fixed cost. In other words, the cost that do not change with the change in output is called fixed cost.

Eg: Rent, salary of permanent staff, interest on loan, insurance premium etc.

COSTS

Total Variable Cost – TVC

The costs incurred by the producer to buy variable inputs is called total variable cost. In other words, the cost that change with the change in quantity of product is called variable cost.

If the quantity of output increased / decreased, variable cost will increase / decrease. If the production become zero, variable cost also become zero. When production becomes positive variable cost also become positive.

Eg: Cost of raw materials, energy cost, salary of temporary employees etc.

COSTS

Total Cost

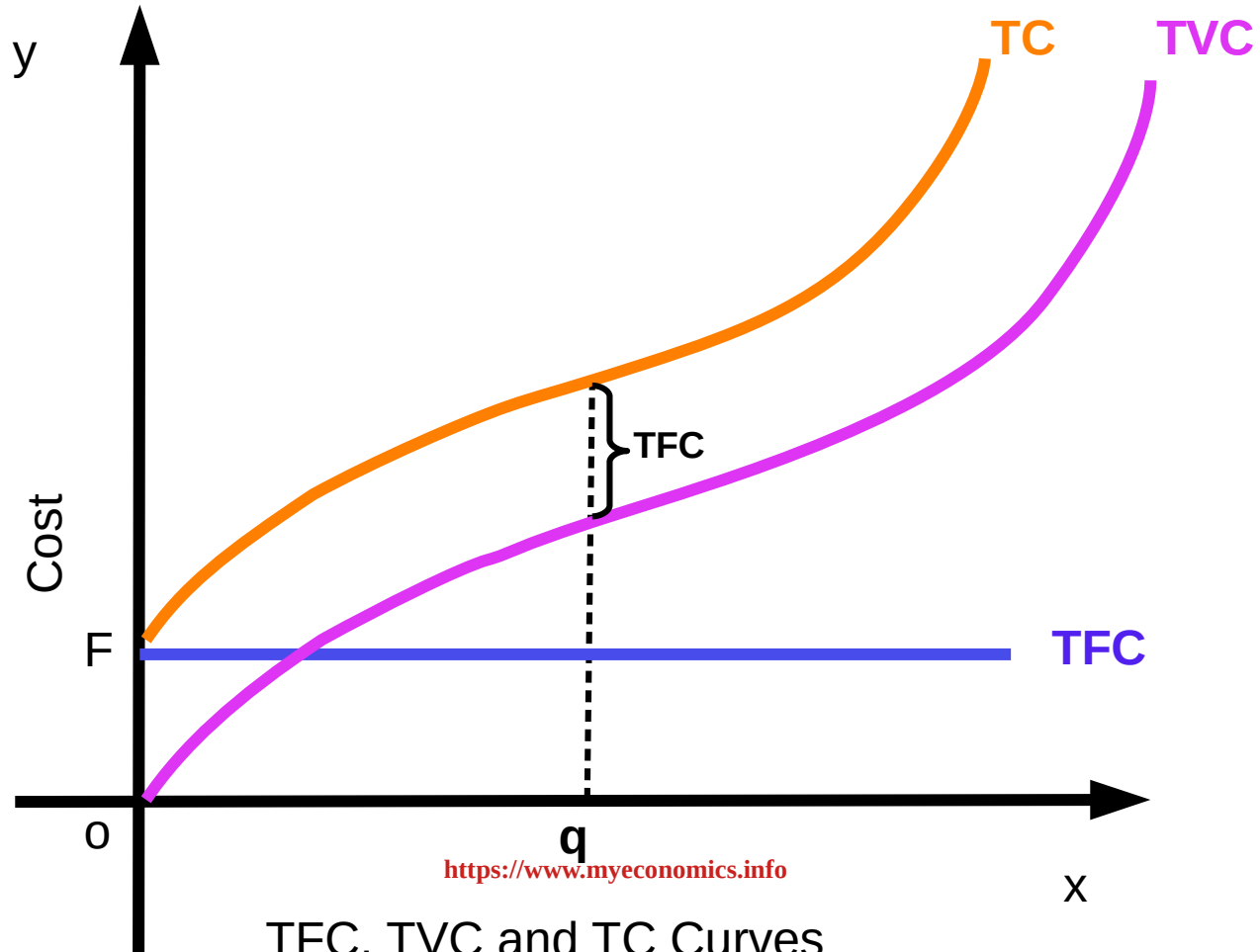
Total Cost is the sum total of all costs incurred in production by the producer to produce goods and services.

$$TC = TFC + TVC$$

$$TFC = TC - TVC$$

$$TVC = TC - TFC$$

Shape of TFC, TVC and TC curves



COSTS

Remember ...

**Difference exists between TFC and TVC only
in the short run.**

COSTS

q	TFC	TVC	TC = TFC + TVC
0	20	0	20
1	20	10	30
2	20	19	39
3	20	27	47
4	20	34	54
5	20	40	60

COSTS

Qn.1) Find TFC and TVC .

q	TC
0	50
1	60
2	70
3	80
4	110
5	150
6	200

COSTS

Ans.

q	TC	TFC	TVC
0	50	50	0
1	60	50	10
2	70	50	20
3	80	50	30
4	110	50	60
5	150	50	100
6	200	50	150

COSTS

Average Variable Cost – AVC



Average Variable Cost is the variable cost per unit of output .

$$AVC = \frac{TVC}{q}$$

$$\therefore TVC = q \times AVC$$

COSTS

- Qn.Find **AVC**

Q	TVC	AVC
0	0	?
1	10	?
2	13	?
3	15	?
4	19	?
5	25	?
6	30	?
7	35	?
8	39	?
9	42	?
10	45	?

COSTS

- An.

Q	TVC	$AVC = \frac{TVC}{q}$
0	0	--
1	10	10
2	13	6.5
3	15	5
4	19	4.75
5	25	5
6	30	6
7	35	5
8	39	4.87
9	42	4.66
10	45	4.5

COSTS

Average Fixed Cost – AFC

Average Fixed Cost is the cost per unit of output.

$$\text{AFC} = \frac{\text{TFC}}{q}$$

$$\therefore \text{TFC} = q \times \text{AFC}$$

COSTS

- Qn. Find AFC

Q	TFC	AFC
0	100	?
1	100	?
2	100	?
3	100	?
4	100	?
5	100	?
6	100	?
7	100	?

COSTS

- An. AFC

Q	TFC	$AFC = \frac{TFC}{Q}$
0	100	--
1	100	100
2	100	50
3	100	33.3
4	100	25
5	100	20
6	100	16.6
7	100	14.2

COSTS

Short Run Average Cost - SAC

In the short run, cost per unit of output is the Short Run Average cost.

$$SAC = \frac{TC}{q}$$

$$SAC = AFC + AVC$$

COSTS

- Qn. Find SAC

q	TC	SAC
0	10	?
1	12	?
2	15	?
3	18	?
4	20	?
5	25	?
6	27	?
7	30	?
8	32	?

COSTS

- An. **SAC**

q	TC	$SAC = \frac{TC}{q}$
0	10	--
1	12	12
2	15	7.5
3	18	6
4	20	5
5	25	5
6	27	4.5
7	30	4.2
8	32	4

COSTS

Short Run Marginal Cost

- **Short Run Marginal cost is the addition to the total cost. It is the change in total cost due to the production of one extra unit of output.**

$$\text{SMC} = \frac{\text{Change in total cost}}{\text{Change in the number of units of output}} = \frac{\Delta \text{TC}}{\Delta q}$$

COSTS

- Qn.Find **SMC**

q	TC	SMC
0	100	?
1	125	?
2	140	?
3	155	?
4	150	?
5	160	?
6	180	?

COSTS

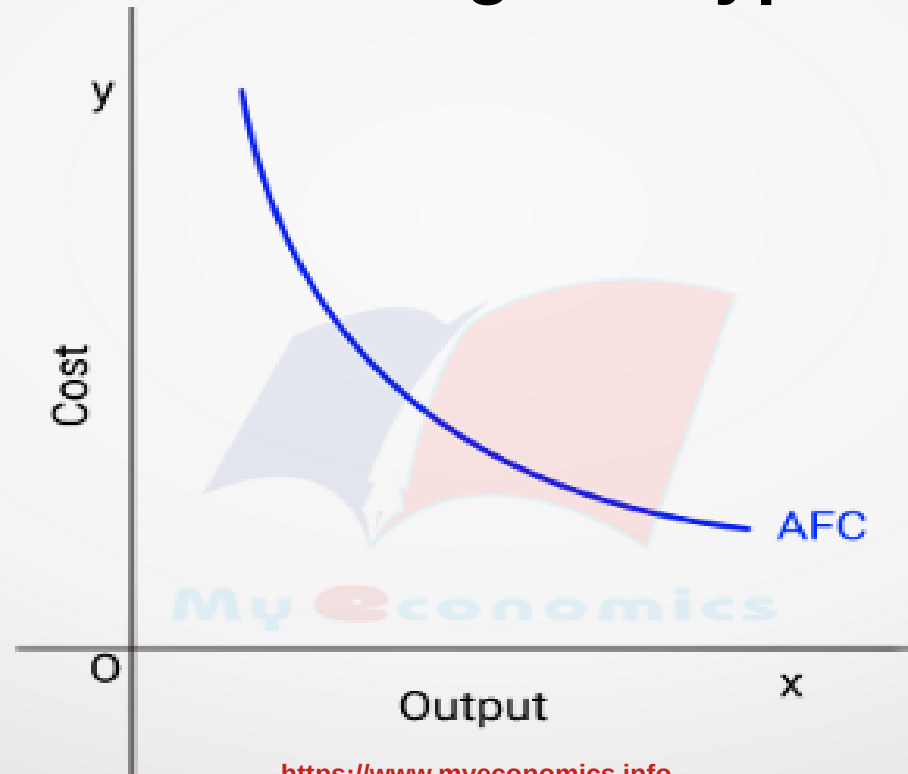
- An. **SMC**

q	TC	$SMC = \frac{\Delta TC}{\Delta q}$
0	100	--
1	125	125
2	140	70
3	155	51.67
4	150	37.5
5	160	32
6	180	30

COSTS

Average Fixed Cost (AFC) Curve

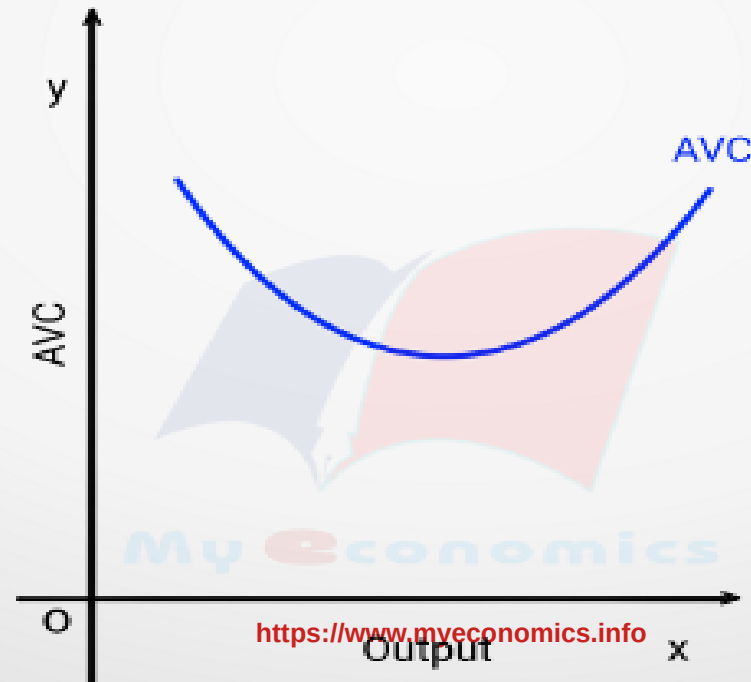
The shape AFC is rectangular hyperbola.



COSTS

Average Variable Cost Curve

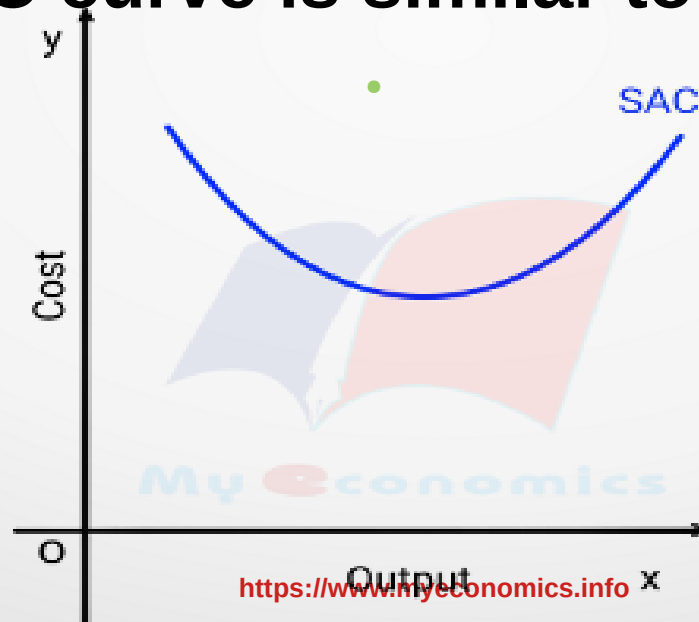
As the quantity of output increases AVC initially decreases and later increases, so the shape of AVC curve is similar to letter 'U'.



COSTS

Short run Average Cost (SAC) Curve

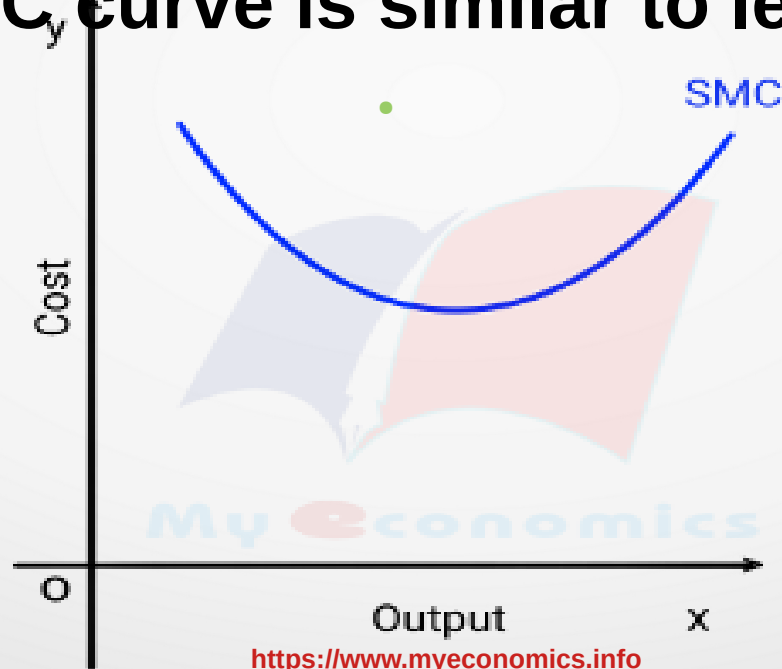
As the quantity of output increases SAC initially decreases and later increases, so the shape of SAC curve is similar to letter 'U'.



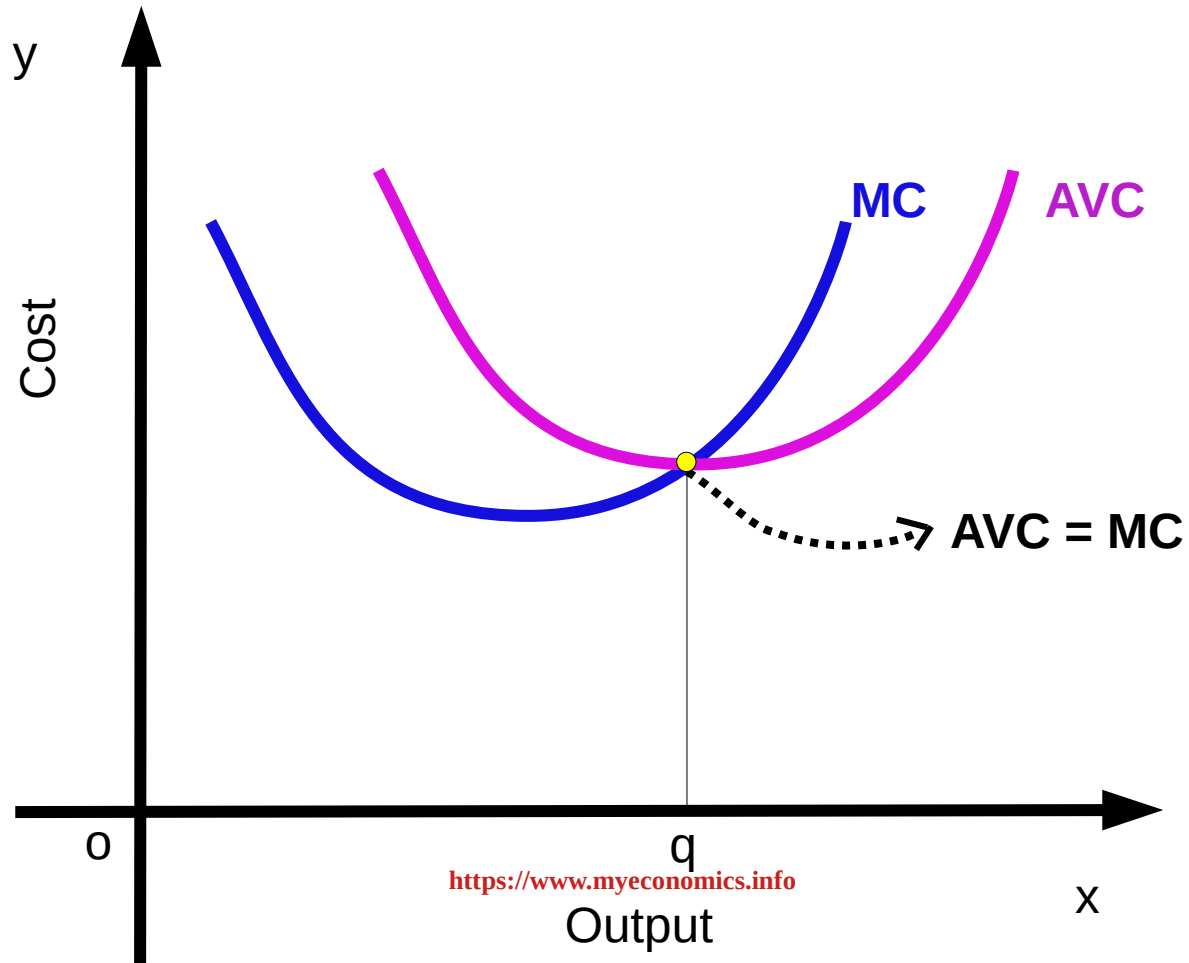
COSTS

Short run Marginal Cost (SMC) Curve

As the quantity of output increases SMC initially decreases and later increases, so the shape of SMC curve is similar to letter 'U'.



The relationship between SAC and SMC or MC:-

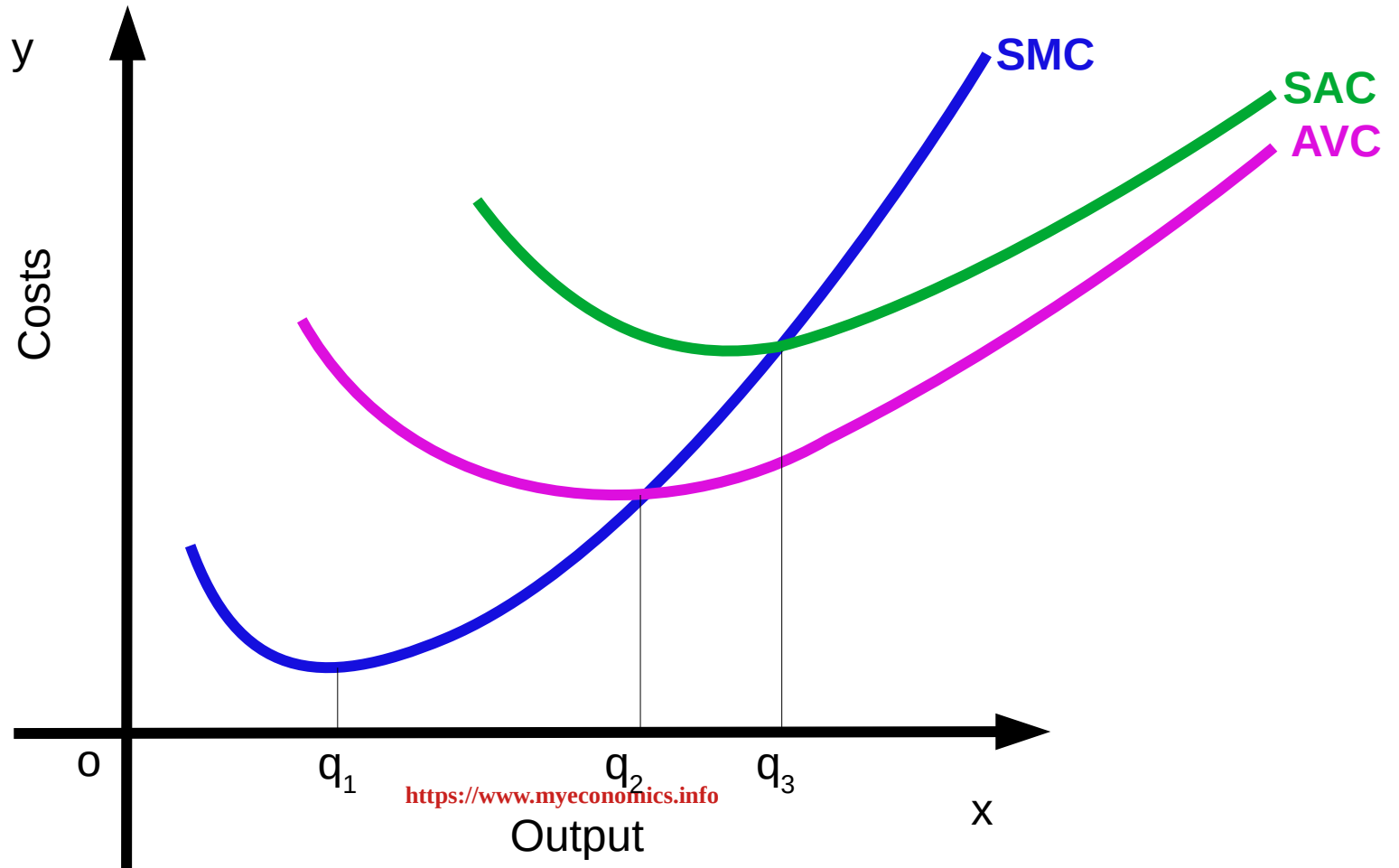


COSTS

The relationship between SAC and SMC or MC:-

- AC and MC initially decrease, but later they increase. So the diagram appears as 'U' shaped.
- When AC decrease, MC will be less than AC.
- **When AC increases, MC will be more than AC.**
- **When AC is minimum, MC and AC will be equal.**

Relationship between SAC curve and AVC curve:-



COSTS

Relationship between SAC curve and AVC curve:-

- * At a particular output the difference between SAC and AVC curves will be equal to the AFC of that output.
- * As the output increases, difference between SAC and AVC decreases.
- * The minimum point of AVC curve will be on the left of the minimum point of SAC.

COSTS

Long Run Costs

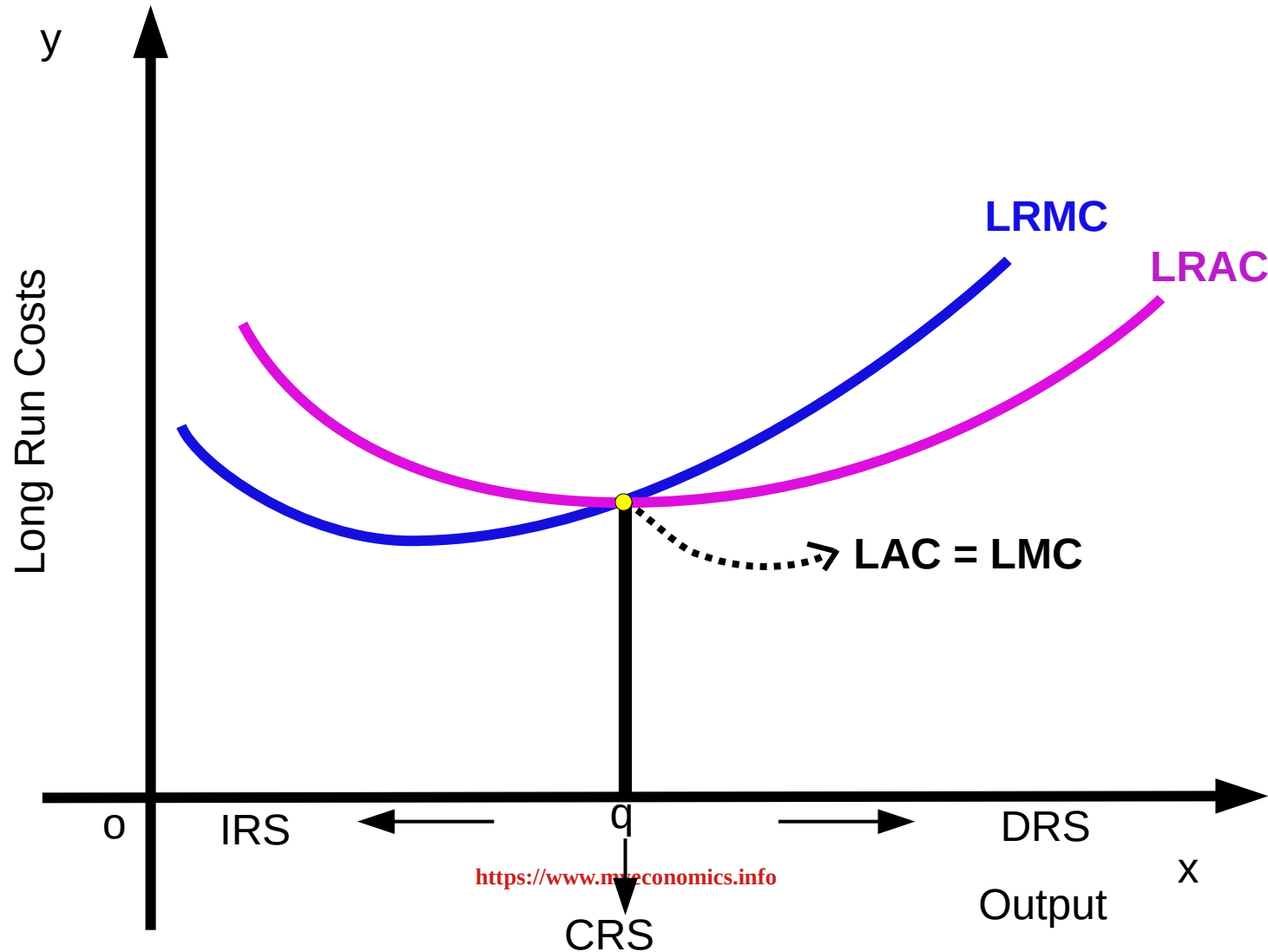
In the long run all inputs are variable. Therefore there is no difference between fixed cost and variable cost. Hence, all costs are variable .

Long Run Average Cost and Long Run Marginal Cost are long run costs.

$$\text{LRAC} = \frac{\text{TC}}{q}$$

$$\text{LRMC} = \frac{\Delta \text{TC}}{\Delta q}$$

Relationship between LRAC and LRMC



Long Run Costs

- **Relationship between LRAC and LRMC**
- As per traditional cost theory LAC and LMC are 'U' shaped.
- When LAC decreases LMC is less than LAC.
When LAC increases LMC is more than LAC.
- LMC curve cuts LAC curve at the minimum point from below.
- Minimum point of LMC will be on the left of LAC. LAC reaches the minimum point first.
- LAC and LMC become equal at the minimum point of LAC.